Solutions

Chapter 4
Playing with Logarithms

Definition:

$\log_a(x) = y$ if and only if $a^y = x$.

2. Evaluate the following logarithms without using a calculator.

- $\log(1,000,000)$
- $\log_5(\frac{1}{25})$
- $\log_6(18) + \log_6(2)$
- $\log_2(200) - \log_2(25)$.

$\log(1,000,000) = \log(10^6) = 6$

$\log_5(\frac{1}{25}) = \log_5(5^{-2}) = -2$

$\log_6(18) + \log_6(2) = \log_6(36) = \log_6(6^2) = 2$

$\log_2(200) - \log_2(25) = \log_2(8) = \log_2(2^3) = 3$

3. What is the domain and range of $\ln(x)$?

$\text{Dom}(\ln(x)) = (0, \infty)$

$\text{Ran}(\ln(x)) = (-\infty, \infty)$

4. Expand each of the following logarithms as far as possible.

- $\log_5(x^3y^6)$ $\log_5(x^3)+\log_5(y^6)=3\log_5(x)+6\log_5(y)$
- $\ln(\frac{ab}{\sqrt{c}})$ $\ln(a)+\ln(b)-\ln(\sqrt{c})=\ln a+\ln b-\frac{1}{2}\ln c$
- $\log(\sqrt[3]{x^2\cdot\frac{z^6}{\sqrt{w}}})$ $\frac{1}{3}\log\left(x^2\cdot\frac{z^6}{\sqrt{w}}\right)=\frac{1}{3}\log(x^2)+\frac{1}{3}\log(z^6)-\frac{1}{3}\log(\sqrt{w})$ $=\frac{2}{3}\log(x)+2\log(z)-\frac{1}{6}\log(w)$

5. Rewrite each expression as a single logarithm.

- $\log(a+b)+\log(a-b)-2\log(c)$ $\log\left(\frac{(a+b)(a-b)}{c^2}\right)=\log\left(\frac{a^2-b^2}{c^2}\right)$
- $4\log_6(y)-\frac{1}{4}\log_6(z)$ $\log_6(y^4)-\log_6(\sqrt[4]{z})=\log_6\left(\frac{y^4}{\sqrt[4]{z}}\right)$
- $\log(5)+2\log(x)+3\log(x^2+5)$ $\log(5)+\log(x^2)+\log((x^2+5)^3)$ $=\log(5x^2(x^2+5)^3)$

Analyzing Tables

Determine if each of the following tables can be modeled by linear function, an exponential function or neither. If the table is linear or exponential, give the function which models the data.

x	y	
0	800	↓400
1	1200	↓480
2	1680	
3	2184	
4	2839	

Not linear

$\frac{1200}{800} = 1.5$, $\frac{1680}{1200} = 1.4$

Not exponential.

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x	y	
0	12,000	↓-1500
3	10,500	↓-1500
6	9,000	↓-1500
9	7,500	↓-1500
12	6,000	

Linear

$\frac{\Delta y}{\Delta x} = \frac{-1500}{3} = -500$

$f(x) = 1200 - 500x$

x	y
4	1400
5	2240
6	3584
7	5734.4
8	9175.04

↓ 840
↓ 1344

Not Linear

$\frac{2240}{1400} = 1.6$, $\frac{3584}{2240} = 1.6$, $\frac{5734.4}{3584} = 1.6$

$\frac{9175.04}{5734.4} = 1.6$

Exponential.

$1400 = C(1.6)^4$ $\qquad f(x) = C(1.6)^x$

$\Downarrow$

$C = \frac{1400}{(1.6)^4} \approx 213.6$

So $f(x) = 213.6(1.6)^x$.